JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 21, No. 6, November–December 1998

Terminal Guidance Algorithm for Ramjet-Powered Missiles

Romesh K. Aggarwal* and Charles R. Moore†
Raytheon Systems Company, Tewksbury, Massachusetts 01876

The terminal guidance design problem for a long-range bank-to-turn (BTT) ramjet missile involves a completely coupled pitch–yaw–roll nonlinear dynamics model. Because of these nonlinear dynamics, an optimal solution for acceleration and roll-rate commands can be found only by numerical methods for solving two-point boundary value problems. Because the roll-rate dynamics of the state-of-the-art missiles is much faster than the rest of the system states, a near-optimal solution is obtained to the BTT guidance problem using multiple timescale techniques. Nonlinear feedback solutions for the acceleration command and roll-rate command are derived. The zero-order slow solution obtained by treating roll rate as infinitely fast, is exactly the optimal guidance law for a skid-to-turn missile. The zero-order fast solution provides the roll-rate command. A first-order analysis corrects the acceleration command for the finite dynamics behavior in bank angle. Simulation results are presented for a representative terminal engagement using this algorithm. Near-zero miss distances are obtained in a noise-free environment.

I. Introduction

RAMJETS or ramjet derivatives are the only viable choice for long-range air-to-air missiles. Ramjets can tolerate very small sideslip angles and negative angles of attack due to propulsion system flameout problems. Thus, ramjet powered missiles utilize bank-to-turn (BTT) guidance. The design of a terminal guidance system for a ramjet BTT missile is a difficult task due to nonlinear pitch–yaw–roll dynamics. BTT configurations were studied for the rocket-powered, short-range air-to-air missiles in Ref. 1, where classical techniques were used to design the guidance system. Reference 2 used singular perturbation techniques to derive BTT guidance law for a short-range air-to-air missile. However, in this formulation the nonlinearity was introduced in the position dynamics due to the heading of the missile velocity vector and the analysis ignored the pitch and roll dynamics.

A long-range missile is essentially on the collision course at the initiation of terminal guidance. Thus, the position dynamics are considered linear, and the nonlinearity is introduced by the roll dynamics of the missile. This problem was treated in Ref. 3, where the system dynamics were linearized about a nominal bank angle and linear quadratic control techniques were utilized to obtain the guidance law. However, the linear solution assumes that missile will not roll through a large angle. This assumption does not hold for many engagements. Reference 4 used multiple timescale techniques to develop a nonlinear BTT guidance law and verified the near optimally of the feedback guidance law by comparing it to the optimal solution. However, this solution was derived assuming perfect (zero-lag) pitch and roll-rate autopilots. The roll-rate time constant of a state-of-the-art missile is very small (0.02–0.04 s). Thus, the roll-rate autopilot can be assumed perfect. However, the time constant of a pitch autopilot is approximately 0.25 s. Thus, pitch-rate dynamics are not negligible compared to roll dynamics. Also, the assumption in Ref. 3 that the missile will not roll through a large angle may be invalid for some engagements.

This paper extends the solution of Ref. 4 to develop a nonlinear feedback guidance law for a BTT missile assuming a first-order pitch autopilot and a zero-lag roll-rate autopilot. As in Ref. 4, the problem is formulated as a cheap control problem, that is, the weighting factor on the roll control cost is chosen small. This weighting factor becomes the perturbation parameter in the multiple timescale analysis, and the resultant problem separates into slow and fast modes. The solution is carried to the first order in the perturbation parameter, which corrects it for the finite roll dynamics. The resultant BTT guidance algorithm was incorporated in a computer program, and a representative set of terminal engagements were simulated. Miss distances were negligible in the absence of noise.

II. Derivation of the BTT Missile Guidance Algorithm

In this section, the terminal guidance problem for a long-range ramjet missile against an air target is formulated as an optimal control problem, and the solution using multiple timescale techniques is outlined.

Problem Formulation

As in Ref. 4, the problem is formulated in a nonrotating coordinate frame that is aligned with the seeker frame at the start of terminal guidance. Figure 1 gives a simple planar representation of the intercept geometry in the x_r, z_r plane, with the x_r axis along the initial line of sight and the z_r axis normal to it. A similar picture applies to the x_r, y_r plane. In this frame, the intercept dynamics for a BTT missile are represented by the following model:

$$\dot{y}_r = V_y \tag{1}$$

$$\dot{V}_y = -A\cos\phi + A_{T_y} \tag{2}$$

$$\dot{z}_r = V_z \tag{3}$$

$$\dot{V}_z = -A\sin\phi + A_{T_z} \tag{4}$$

$$\dot{A} = (A_c - A)/\tau_a \tag{5}$$

$$\dot{\phi} = \dot{\phi}_c \tag{6}$$

where y_r and z_r are the relative position errors, V_y and V_z are the relative velocity errors, $A\cos\phi$, $A\sin\phi$, A_{T_y}, and A_{T_z} are the respective components of missile and target accelerations normal to the line of sight, and ϕ is the missile roll angle. A_c and $\dot{\phi}_c$ are, respectively, the missile acceleration command and roll-rate command. The missile is assumed to have a zero-lag roll-rate autopilot and a first-order acceleration autopilot with time constant τ_a. The closing velocity V_c along the initial line of sight x_r is assumed constant. Thus, the intercept time is obtained from the relation

$$x_r = V_c(t_f - t) \tag{7}$$

An optimal choice for the controls A_C and $\dot{\phi}_C$ is sought, where the criterion for the optimality is the minimization of the performance index given by

$$J = \frac{1}{2}\left\{ y_r^2(t_f) + z_r^2(t_f) + \int_{t_0}^{t_f} \left(b_a A_c^2 + b_{\dot{\phi}}\dot{\phi}_c^2\right) \mathrm{d}t \right\} \tag{8}$$

Presented as Paper 96-3738 at the AIAA Guidance, Navigation, and Control Conference, San Diego, CA, July 29–31, 1996; received Jan. 9, 1997; revision received June 16, 1997; accepted for publication March 5, 1998.

*Project Engineer, Systems Design Laboratory. Member AIAA.
†Project Engineer, Systems Design Laboratory.

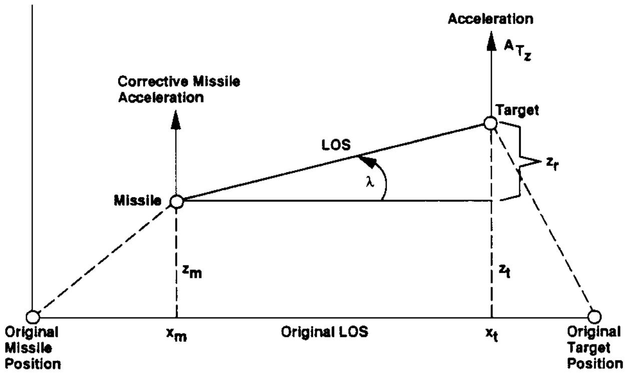

Fig. 1 Moving nonrotating coordinate system for air-to-air missile.

Optimal Solution

The Lagrange multiplier method may be used to find the optimal solution.[5] The Hamiltonian for this problem is

$$H = \lambda_y V_y + \lambda_{V_y}\left(-A\cos\phi + A_{T_y}\right) + \lambda_z V_z + \lambda_{V_z}\left(-A\sin\phi + A_{T_z}\right) + \lambda_\phi\dot{\phi}_c + \tfrac{1}{2}\left(b_a A_c^2 + b_{\dot{\phi}}\dot{\phi}_c^2\right) + \lambda_A(A_c - A)/\tau_a \quad (9)$$

Because the Hamiltonian does not depend on y_r or z_r,

$$\dot{\lambda}_y = 0 \quad (10)$$

$$\dot{\lambda}_z = 0 \quad (11)$$

The other four Lagrange variables are

$$\dot{\lambda}_{V_y} = -\frac{\partial H}{\partial V_y} = -\lambda_y \quad (12)$$

$$\dot{\lambda}_{V_z} = -\frac{\partial H}{\partial V_z} = -\lambda_z \quad (13)$$

$$\dot{\lambda}_A = -\frac{\partial H}{\partial A} = \lambda_{V_y}\cos\phi + \lambda_{V_z}\sin\phi + \frac{\lambda_A}{\tau_a} \quad (14)$$

$$\dot{\lambda}_\phi = -\frac{\partial H}{\partial \phi} = A\left(\lambda_{V_z}\cos\phi - \lambda_{V_v}\sin\phi\right) \quad (15)$$

The final values of λ_{V_y}, λ_{V_z}, λ_A, and λ_ϕ are zero and

$$\lambda_y(t_f) = y_r(t_f) \quad (16)$$

$$\lambda_z(t_f) = z_r(t_f) \quad (17)$$

Define

$$y_r(t_f) = C\cos\bar{\phi} \quad (18)$$

$$z_r(t_f) = C\sin\bar{\phi} \quad (19)$$

where C is the unknown terminal miss distance given by

$$C = \sqrt{y_r^2(t_f) + z_r^2(t_f)} \quad (20)$$

and

$$\bar{\phi} = \tan^{-1}\frac{z_r(t_f)}{y_r(t_f)} \quad (21)$$

with these definitions, Eqs. (10–13), with terminal values of λ_{V_y} and λ_{V_z} set to zero, result in the following:

$$\lambda_y(t) = \text{const} = C\cos\bar{\phi} \quad (22)$$

$$\lambda_z(t) = \text{const} = C\sin\bar{\phi} \quad (23)$$

$$\lambda_{V_y}(t) = C\cos\bar{\phi}(t_f - t) \quad (24)$$

$$\lambda_{V_z}(t) = C\sin\bar{\phi}(t_f - t) \quad (25)$$

and Eqs. (14) and (15) become

$$\dot{\lambda}_A = C[\cos(\bar{\phi} - \phi)](t_f - t) + \lambda_A/\tau_a, \qquad \lambda_A(t_f) = 0 \quad (26)$$

$$\dot{\lambda}_\phi = AC[\sin(\bar{\phi} - \phi)](t_f - t), \qquad \lambda_\phi(t_f) = 0 \quad (27)$$

The optimality conditions give optimal acceleration and roll-rate commands

$$\frac{\partial H}{\partial A_c} = 0 = \frac{\lambda_A}{\tau_a} + b_a A_c \quad (28)$$

and

$$\frac{\partial H}{\partial \dot{\phi}_c} = 0 = \lambda_\phi + b_{\dot{\phi}}\dot{\phi}_c \quad (29)$$

Note that the given optimal control problem is a two-point boundary value problem (TPBVP) involving Eqs. (1–8), (18), (19), and (26–29). Several numerical methods are available for the solution of the TPBVP. Although real-time solution of the TPBVP is possible, the numerical techniques are computationally intensive and may strain onboard throughput requirements. The following develops a near-optimal feedback solution of this problem using the multiple timescale analysis based on perturbation techniques.

The final closed-form solution is a simple function of existing state variables, which can be readily implemented for real-time operation. System performance could then be compared between a more simple BTT law and the first-order solution provided here to determine the degree of performance improvement. If further improvement were required, the exact solution of the TPBVP could be implemented.

Perturbation Solution

Perturbation methods can be described as model order reduction techniques that capitalize on the presence of slow and fast modes in the system. These methods have been applied to problems in optimal control and estimation.[6,7] In linear problems, ill conditioning in the dynamics is avoided, and a high-order problem is reduced to the solution of two (or more) low-order problems. In nonlinear problems, perturbation techniques have led to nonlinear control solutions in a feedback form.

In the perturbation techniques, the optimal control and trajectory solution is expanded in an outer expansion solution about a system parameter $\varepsilon = 0$:

$$\cdot(t,\varepsilon) = \cdot(t,0) + \frac{\partial\cdot}{\partial\varepsilon}\varepsilon + \cdots, \qquad t \in (t_0, t_f) \quad (30)$$

where $\cdot$ denotes the states, adjoints, and control variables. However, the expansion in Eq. (30) is not uniformly valid in the interval $t_0 \le t \le t_f$ because the fast states and their adjoints will not satisfy their respective boundary conditions at initial and/or terminal times. This leads to the occurrence of boundary layer or inner expansion solutions

$$\cdot(\tau,\varepsilon) = \cdot(\tau,0) + \frac{\partial\cdot}{\partial\varepsilon}\varepsilon + \cdots, \qquad t = t_0 \quad (31)$$

$$\cdot(\sigma,\varepsilon) = \cdot(\sigma,0) + \frac{\partial\cdot}{\partial\varepsilon}\varepsilon + \cdots, \qquad t = t_f \quad (32)$$

These solutions are derived using the time stretching transformations

$$\tau = (t - t_0)/\varepsilon, \qquad \sigma = (t_f - t)/\varepsilon \quad (33)$$

about $t = t_0$ and $t = t_f$, respectively. The total solution, which is the sum of inner and outer expansions, satisfies all of the boundary conditions and is uniformly valid in the closed interval to $t_0 \le t \le t_f$.

The inner expansion solution at terminal time is only required in cases where fast states have specified terminal values. For the BTT problem, bank angle is the fast state, and because the bank angle is not specified at terminal time t_f, in this case no inner expansion is required at terminal time.

Zero-Order Outer Solution

From Eqs. (8) and (29), the optimality conditions, it can be seen that when $b_{\dot\phi} \to 0$ ($\dot\phi_c$ is a cheap control) the optimum $\dot\phi_c$ at initial time t_0 is an impulse. Moreover, Eqs. (27) and (29) imply that this roll-rate command impulse at time t_0 will bring the roll angle ϕ to $\bar\phi$ (value of optimum ϕ will be determined later) and for time $t > t_0$

$$\lambda_\phi(t) = 0, \qquad (t_0, t_f] \tag{34}$$

$$\dot\lambda_\phi(t) = 0, \qquad (t_0, t_f] \tag{35}$$

Thus, the system dynamics are separated into slow and fast modes, with relative position, velocity, and pitch acceleration being slower and bank-angle dynamics being faster. Thus, the zero-order slow solution is obtained as follows:

$$\phi^0 = \bar\phi \tag{36}$$

where superscript 0 refers to zero-order outer solution terms. Equations (26) and (28) provide

$$A_c^0 = (C/b_a)\big((t_f - t) + \tau_a\{\exp[-(t_f - t)/\tau_a] - 1\}\big) \tag{37}$$

Substituting these values of ϕ^0 and A_c^0 in Eqs. (2) and (5), integrating, and using Eq. (18), we obtain

$$A_c^0(t_0)\cos\bar\phi = \frac{N'}{(t_f - t_0)^2}\left[y_r + V_y(t_f - t_0) + \frac{1}{2}A_{T_y}(t_f - t_0)^2\right] - N'A(t_0)\cos\bar\phi\frac{(T + e^{-T} - 1)}{T^2} \tag{38}$$

where

$$T = \frac{(t_f - t_0)}{\tau_a} \tag{39}$$

and

$$N' = \frac{6T^2[T + e^{-T} - 1]}{\left[3 + 6T - 6T^2 + 2T^3 - 12Te^{-T} - 3e^{-2T} + 6b_a\big/\tau_a^3\right]} \tag{40}$$

Integrating Eqs. (3) and (4), a similar expression can be obtained:

$$A_c^0(t_0)\sin\bar\phi = \frac{N'}{(t_f - t_0)^2}\left[z_r + V_z(t_f - t_0) + \frac{1}{2}A_{T_z}(t_f - t_0)^2\right] - N'A(t_0)\sin\bar\phi\frac{(T + e^{-T} - 1)}{T^2} \tag{41}$$

Note that this zero-order outer solution is exactly the optimal guidance law obtained by applying conventional linear control theory to the skid-to-turn (STT) missile guidance problem.

Considering current time as the initial time, we obtained the feedback form of the solution. Thus, the zero-order outer solution is

$$\bar\phi = \tan^{-1}\left[\frac{\left\{z_r + V_z(t_f - t_0) + \frac{1}{2}A_{T_z}(t_f - t_0)^2\right\}}{\left\{y_r + V_y(t_f - t_0) + \frac{1}{2}A_{T_y}(t_f - t_0)^2\right\}}\right] \tag{42}$$

and

$$A_c^0(t_0) = \frac{N'}{(t_f - t_0)^2}\left[\left\{y_r + V_y(t_f - t_0) + \frac{A_{T_y}}{2}(t_f - t_0)^2\right\}^2 + \left\{z_r + V_z(t_f - t_0) + \frac{A_{T_z}}{2}(t_f - t_0)^2\right\}^2\right]^{\frac{1}{2}} - N'A(t_0)\frac{[T + e^{-T} - 1]}{T^2} \tag{43}$$

$$H^0 = \lambda_y^0 V_y + \lambda_{V_y}^0 V_y\big(-A\cos\bar\phi + A_{T_y}\big) + \lambda_z^0 V_z + \lambda_{V_z}^0 V_z\big(-A\sin\bar\phi + A_{T_z}\big) + \lambda_A^0\big(A_c^0 - A\big)\big/\tau_a + \tfrac{1}{2}b_a A_c^{02} \tag{44}$$

Zero-Order Inner Solution

As shown earlier for BTT problem, the relative position, velocity, and pitch acceleration are slow states and the bank angle is the fast state. Slow states and adjoints remain constant in zero-order inner expansion.

Incorporating the time stretching transformation $\tau = (t - t_0)/b_{\dot\phi}$ results in the boundary-layer equations for the missile bank angle transition at the initiation of terminal guidance. Letting $b_{\dot\phi} \to 0$ results in the following set of necessary conditions:

$$y_{r_1}^0(\tau) = y_r(t_0), \qquad \lambda_{y_1}^0(\tau) = \lambda_y^0 \tag{45}$$

$$z_{r_1}^0(\tau) = z_r(t_0), \qquad \lambda_{z_1}^0(\tau) = \lambda_z^0 \tag{46}$$

$$V_{y_1}^0(\tau) = V_y(t_0), \qquad \lambda_{V_{y_1}}^0(\tau) = \lambda_{V_y}^0(t_0) \tag{47}$$

$$V_{z_1}^0(\tau) = V_z(t_0), \qquad \lambda_{V_{z_1}}^0(\tau) = \lambda_{V_z}^0(t_0) \tag{48}$$

$$A^0(\tau) = A(t_0), \qquad \lambda_{A1}^0(\tau) = \lambda_A^0(t_0) \tag{49}$$

$$\frac{d\phi_1^0}{d\tau} = b_{\dot\phi}\dot\phi_c, \qquad \phi_1^0(0) = \phi(t_0), \qquad \phi_1^0(\infty) = \bar\phi \tag{50}$$

$$H_1^0 = \lambda_y^0 V_y(t_0) + \lambda_z^0 V_z(t_0) + \lambda_{V_y}^0(t_0)\big(-A(t_0)\cos\phi + A_{T_y}\big) + \lambda_{V_z}^0(t_0)\big(-A(t_0)\sin\phi + A_{T_z}\big) + \lambda_{\phi_1}^0\dot\phi_c + \tfrac{1}{2}b_a A_{c_i}^{02} + \tfrac{1}{2}b_{\dot\phi}\dot\phi_c^2 + \lambda_A^0\big(A_{c_1}^0(t_0) - A(t_0)\big)\big/\tau_a \tag{51}$$

where superscript 0 indicates zero-order solution and subscript 1 indicates inner solution. Thus, the combination stands for zero-order inner solution. Optimality conditions give

$$\frac{\partial H_{1^0}}{\partial A_{c_1}^0} = \frac{\lambda_A^0}{\tau_a} + b_a A_{c_1}^0 = 0, \qquad A_{c_1}^0 = -\frac{\lambda_A^0}{b_a\tau_a} = A_c^0 \tag{52}$$

$$\frac{\partial H_1^0}{\partial\dot\phi_c} = \lambda_{\phi_1}^0 + b_{\dot\phi}\dot\phi_c = 0, \qquad \lambda_{\phi_1}^0 = -b_{\dot\phi}\dot\phi_c \tag{53}$$

Because the Hamiltonian remains constant for an autonomous system, H_1^0 is equal to H^0, and from Eqs. (44), (51), and (53) we obtain

$$-\lambda_{\phi_1}^0\dot\phi_c = 2A(t_0)\big[\lambda_{V_y}^0(t_0)(\cos\bar\phi - \cos\phi) + \lambda_{V_z}^0(t_0)(\sin\bar\phi - \sin\phi)\big] = 2A(t_0)C(t_f - t_0)[1 - \cos(\bar\phi - \phi)] \tag{54}$$

Using Eqs. (37), (53), and (54) we obtain

$$\dot\phi_c = \sqrt{\frac{2b_a T A(t_0)\cdot A_c^0(t_0)[1 - \cos(\bar\phi - \phi)]}{b_{\dot\phi}(T + e^{-T} - 1)}} \times \operatorname{sign}(\bar\phi - \phi) \tag{55}$$

This zero-order inner solution provides the roll-rate command.

First-Order Solution

Note that zero-order solution for acceleration command assumes the missile has achieved the optimum bank angle instantly at initial time t_0. However, because of finite roll rate, the missile can not attain the optimum bank angle instantly, and the zero-order acceleration command must be corrected to compensate for finite roll dynamics.

If we assume that the states are measurable and the control solution is updated as a function of the current state, then only the first-order correction in λ_A is required to correct the acceleration command. Because the system dynamics are independent of y_r and z_r, the solution is not sensitive to first-order correction in λ_y, λ_z, λ_{Vy}, and λ_{Vz}. Thus, the acceleration command to the first order is given by

$$A_c = -\frac{\big(\lambda_{A_0}^0 + \lambda_{A_1}^1\big)}{b_a\tau_a} \tag{56}$$

The first-order correction in λ_A is obtained next by following the procedure outlined in Refs. 8 and 9, which involves matching between the outer solution and the first boundary layer.

From the matching conditions, we have

$$\lambda^1_{A_1}(0) = \lambda^1_{A_0} - \int_0^{\tau*} \frac{\mathrm{d}}{\mathrm{d}\tau}\lambda^1_{A_1}(\tau)\,\mathrm{d}\tau + \tau*\frac{\mathrm{d}}{\mathrm{d}\tau}\lambda^1_{A_1}(\tau*) \tag{57}$$

where

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\lambda^1_{A_1} = -\frac{\partial H_1}{\partial A}\bigg|^0 = \lambda^0_{V_y}\cos\phi + \lambda^0_{V_z}\sin\phi + \frac{\lambda^0_{A_1}}{\tau_a} \tag{58}$$

The time τ^* is picked sufficiently large so that $\phi \approx \bar{\phi}$. Thus,

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\lambda^1_{A_1}(\tau*) = \lambda^0_{V_y}\cos\bar{\phi} + \lambda^0_{V_z}\sin\bar{\phi} + \frac{\lambda^0_{A_1}}{\tau_a} \tag{59}$$

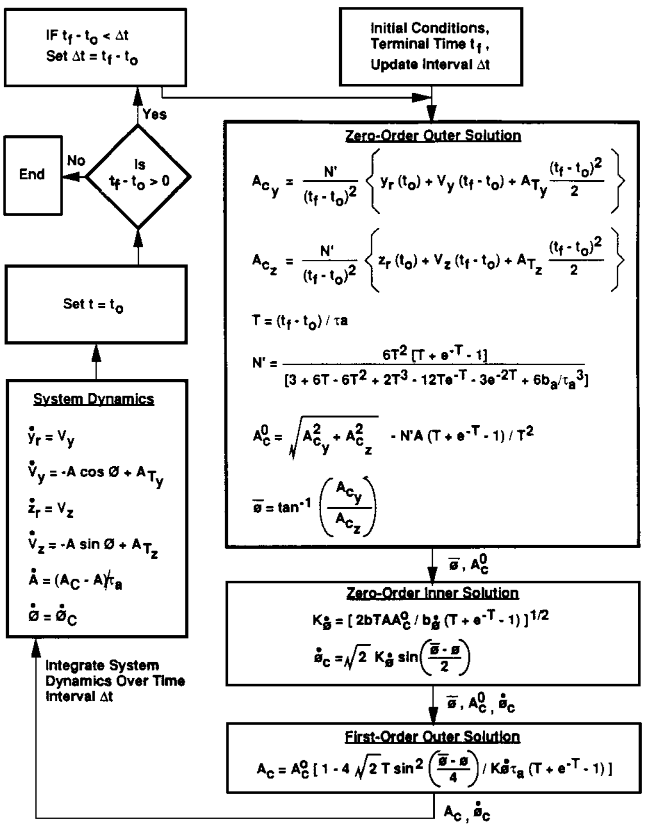

Fig. 2 Sequence of perturbation solution computations.

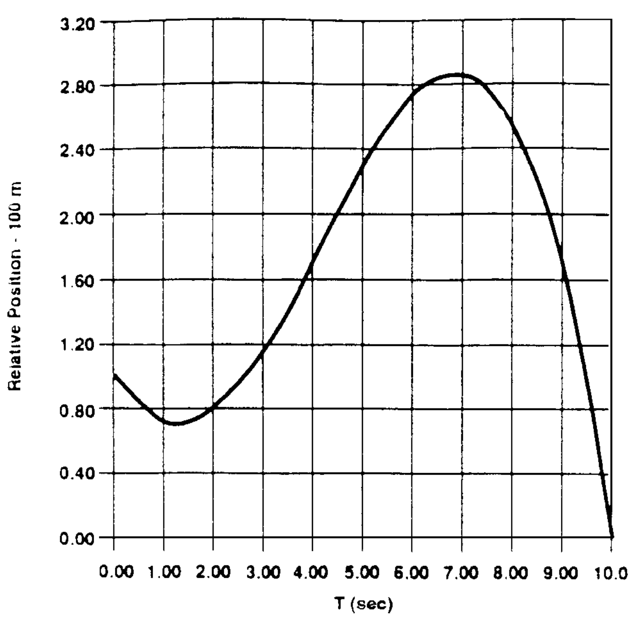

Fig. 3 Relative position histories.

Also $\lambda^1_{A_0}$ is equal to zero. Thus, using Eqs. (57) and (59), the first-order correction in λ_A is given by

$$\lambda^1_{A_1}(0) = \int_0^{\tau*} C(t_f - t_0)[1 - \cos(\bar{\phi} - \phi)]\,\mathrm{d}\tau \tag{60}$$

From Eq. (55)

$$\dot{\phi} = K_{\dot{\phi}}[1 - \cos(\bar{\phi} - \phi)]^{\frac{1}{2}} = \sqrt{2}K_{\dot{\phi}}\sin[(\bar{\phi} - \phi)/2]$$

$$K_{\dot{\phi}} = \sqrt{\frac{2b_a T A(t_0) A^0_c(t_0)}{b_{\dot{\phi}}(T + e^{-T} - 1)}} \tag{61}$$

Now assuming $K_{\dot{\phi}} \neq 0$, Eq. (60) can be written as

$$\lambda^1_{A_1}(0) = \left[\frac{C(t_f - t_0)}{K\dot{\phi}}\right]\int_{\phi_c}^{\bar{\phi}}[1 - \cos(\bar{\phi} - \phi)]^{\frac{1}{2}}\,\mathrm{d}\phi \tag{62}$$

$$\lambda^1_{A_1}(0) = 2\sqrt{2}\left[\frac{C(t_f - t_0)}{K\dot{\phi}}\right]\left[1 - \cos\left(\frac{\bar{\phi} - \phi_0}{2}\right)\right] \tag{63}$$

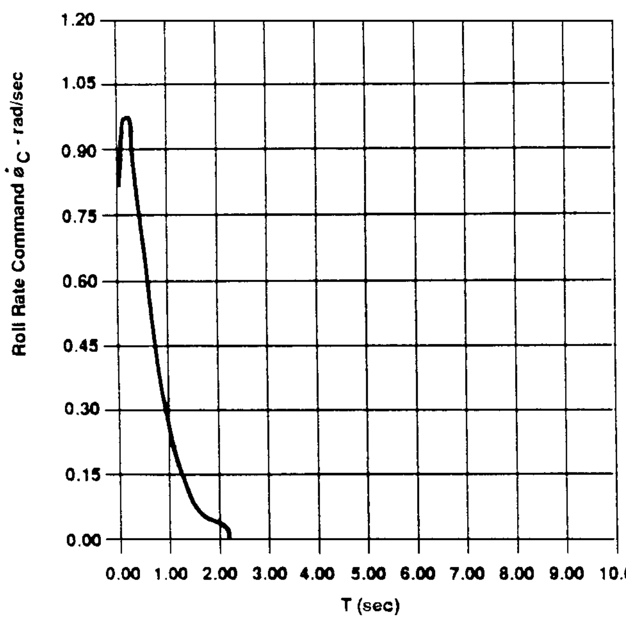

Fig. 4 Roll-rate command history.

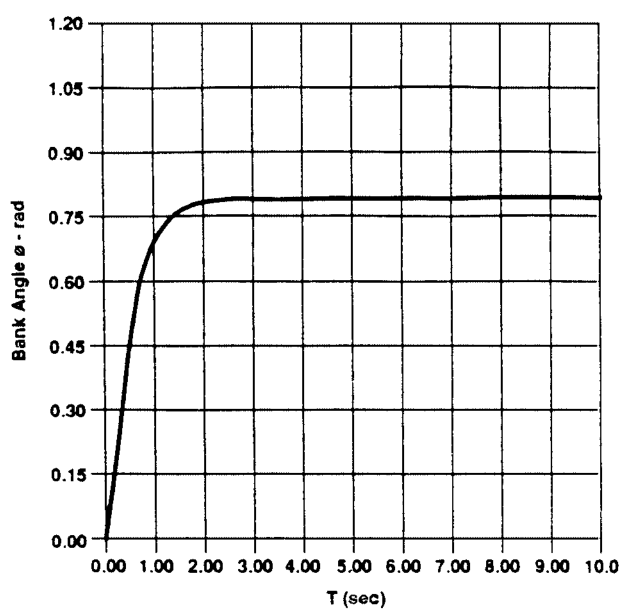

Fig. 5 Missile bank angle history.

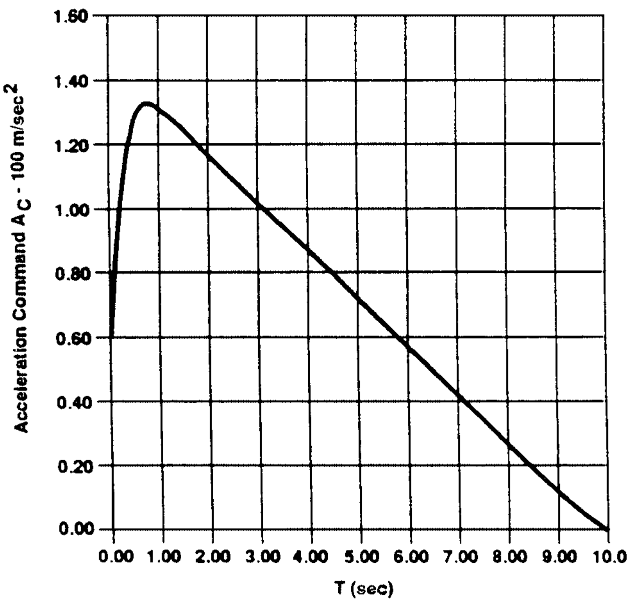

Fig. 6 Acceleration command history.

Thus, using Eqs. (37), (52), (56), and (63), the acceleration command is

$$A_c = A_c^0\left[1 - \frac{2\sqrt{2}T}{K_{\dot\phi}\tau_a(T + e^{-T} - 1)}\left\{1 - \cos\left(\frac{\bar\phi - \phi_0}{2}\right)\right\}\right]$$

$$= A_c^0\left[1 - \frac{4\sqrt{2}T\,\sin^2[(\bar\phi - \phi_0)/4]}{K\dot\phi\tau_a(T + e^{-T} - 1)}\right] \tag{64}$$

Because the time constant of the roll-rate autopilot is very small (0.02–0.04 s) and the guidance solution is the updated onboard missile at 80–100 Hz rate, no higher-order terms are required for the roll-rate command solution.

III. Numerical Results

Perturbation solution trajectories were generated by numerically integrating Eqs. (1–6). The acceleration command and roll-rate command for the BTT missile are defined by the outer and inner solutions discussed earlier given by Eqs. (39), (40), (42), (43), (61), and (64). These commands are updated at each integration step based on the current relative position, velocity, and target acceleration and are applied for the next step. Thus, the state vector after each integration step is used as a new initial condition for the optimization loop. The sequence of computation is shown in Fig. 2.

A number of engagements were simulated. Figures 3–6 show the relative position, missile roll rate, missile bank angle, and acceleration command histories for a typical engagement. The target was making a 9-g maneuver at the initiation of terminal engagement. Near optimality of the solution was illustrated in Ref. 4 for a zero-time-lag pitch autopilot case by comparison with the optimal solution.

IV. Summary

Perturbation techniques have been used to develop a nonlinear feedback guidance algorithm for a BTT missile. The remarkable similarity of the algorithms to STT guidance algorithms used in existing missiles, the same filter requirements, and the good computational efficiency make these algorithms very attractive for real-time, onboard implementation.

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